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The Uniform Expansion of the Universe

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Abstract—The standard equations of general relativity admit extension so that they can be supplemented, not only with Einsteinian cosmological repulsive forces described by the Λ term, but also with other forces. Accordingly, we suggest a model of a uniformly expanding Universe (an S model). In this model, the cosmological forces of attraction and repulsion precisely balance each other. This S model is a good approximation for describing the Universe's evolution over a wide range of redshifts (up to $z \sim 1000$). The S model can explain in a simple way observational data on the age of the Universe, the apparent magnitude—redshift relation for Type Ia supernovae, and the angular separation between the centers of neighboring bright spots against the uniform background of the cosmic microwave background radiation.

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1. INTRODUCTION

Comparatively recently, it was supposed that the dynamics of the Universe were determined by attractive forces (see, e.g., [1, 2]), whereas they are now believed to be determined by both attractive and repulsive forces. The first clear indication of this was probably observational data on the dependence between the apparent magnitudes m and redshifts for Type Ia supernovae [3, 4].

In modern cosmology, it is supposed that these data cannot be explained in the framework of general relativity (GR) without invoking a cosmological constant describing repulsive forces, which play an important role in the dynamics of the Universe (see, e.g., [5—7]).

The radius of curvature (scale factor) $a(t)$ characterizes the dynamics of a a homogeneous and isotropic Universe. The Friedmann cosmological equations describe the time evolution of $a(t)$. These equations follow from the Einstein GR equations and the assumption that the Universe is homogeneous and isotropic. Details concerning the Friedmann cosmological equations and the methodology for their derivation from the Einstein equations can be found, e.g., in [1, 2, 7].

One modern cosmological model based on the Friedmann equations with a cosmological constant is the ΛCDM (Cold Dark Matter) model. In this model, the greater the effect of the Einsteinian repulsive forces (the cosmological constant), the farther objects with a given redshift z are located, and the lower their apparent brightness (the larger their apparent magnitude). Agreement between the apparent magnitude—redshift relation calculated theoretically for the ΛCDM model and the corresponding observed dependence is achieved if the repulsive forces are taken to be much more effective in the modern Universe compared to attractive forces. It is this circumstance that has led to the statement that the cosmological constant has an important role in the accelerated expansion of the Universe.

One source of cosmological repulsive forces described by the Λ term in the Einstein equations is designated in modern cosmology by the term "dark energy." It is believed that dark energy is a certain vacuum-like medium. The Λ term in the GR equations gives a description of its macroscopic properties (see, e.g., [5—7]). Dark energy is thought to be a perfectly homogeneous medium with a density that is constant in time and space in all reference frames:

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}, \tag{1}$$

where G is the gravitational constant, c the velocity of light, Λ the cosmological constant, and $\Lambda > 0$. Here and below, the subscript Λ denotes quantities calculated in the ΛCDM model. Dark energy has a negative pressure. Its equation of state has the

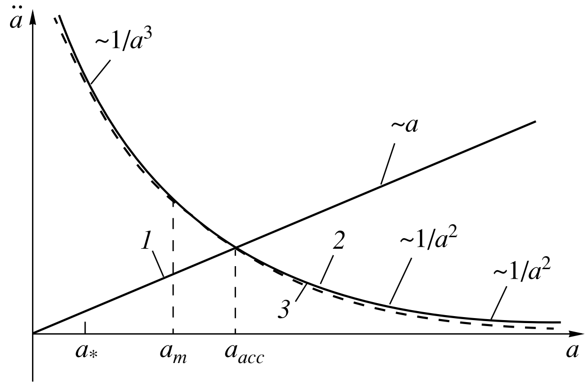

Fig. 1. Nature of the scale-factor dependence for (*1*) Einsteinian repulsive forces in the ΛCDM model, (*2*) attractive forces in the ΛCDM and S models, and (*3*) repulsive forces in the S model.

form [6]

$$P_\Lambda = -\varepsilon_\Lambda, \tag{2}$$

where $\varepsilon_\Lambda = \rho_\Lambda c^2$.

According to the GR equations, the cosmological acceleration due to the cosmological constant is

$$\ddot{a}_\Lambda = -\frac{4}{3}\pi G\left(\rho_\Lambda + \frac{3P_\Lambda}{c^2}\right)a \tag{3}$$

(see, e.g., [1, Ch. 4]). Due to (2), the quantity $\rho_\Lambda + 3P_\Lambda/c^2 < 0$, and therefore $\ddot{a}_\Lambda > 0$. This means that dark energy with the equation of state (2) is a source of repulsive forces.

Besides the Einsteinian repulsive forces, other forms of cosmological repulsive forces are also discussed in the literature. In these variants, as is the case with the cosmological constant, the repulsive forces are taken to be associated with media with negative pressures. The equation of state of these media is taken in the form

$$P = w\rho c^2. \tag{4}$$

For "quintessence" [8–12], the parameter w is taken to obey the condition $-1 < w < -1/3$. In this case,

$$\rho + 3P/c^2 < 0. \tag{5}$$

For this reason, as in the case (3), the cosmological acceleration created by the quintessence is positive, and it is a source of repulsive forces.

Another hypothesis connects the repulsive forces with "phantom energy" [13–15]. The equation of state for this idealized medium has the form (4), but the parameter w obeys the condition $w < -1$.

The growing precision of cosmological observations narrows the range of w in (4) for which models of negative-pressure media agree with the observations. According to the data of [16–18],

$$w = P/\rho c^2 = -0.97 \pm 0.09. \tag{6}$$

It is believed that the substantial reduction of the range of admissible values of w and its proximity to -1 provide strong arguments supporting the view that the cosmological forces are Einsteinian. This opinion is currently the most widespread, and is even believed to have almost been proven (see, e.g., [6, 7, 16–20]).

According to the ΛCDM model, attractive forces played a decisive role in the early Universe. As the Universe expanded, the role of such forces decreased. Meanwhile, the role of the Einsteinian repulsive forces, which grow linearly as the Universe grows in size, increased more and more. The relationship between the attractive and repulsive cosmological forces varies over a wide range (Fig. 1). According to the ΛCDM model, the expansion of the Universe is highly non-uniform. In this model, the functions $\dot{a}(t)$ and $\ddot{a}(t)$ grow in an unbounded fashion as $a(t) \to 0$ and $a(t) \to \infty$.

One shortcoming of the repulsive-forces explanation based on the cosmological constant is the lack of understanding of the physical properties of the entity it describes. This is connected with a fundamental difficulty in the description of the physical properties of dark energy in the framework of known theories.

Klimenko and Fridman [21] consider a fundamentally different explanation of the cosmological repulsive forces that is not based on the cosmological constant. They show that the Einstein version of repulsive forces in GR is not the only one possible, and suggest the introduction of non-Einsteinian repulsive forces into the GR equations. They emphasize that the Friedmann standard cosmological equations do not take into account the possibility of increasing the kinetic energy of the expanding cosmic medium due to its thermal energy decrease. Bearing this in mind, they put forward the hypothesis that the kinetic energy of the expansion and the medium's thermal energy could be described in these equations in a similar way, and that the thermal energy of the cosmic medium is not only that due to sources of gravitational field, which had previously been taken into account, but also due to a source of repulsive forces.

When realized in a consistent way in the Friedmann equations, the idea of a symmetric description of the kinetic energy of the expansion and the thermal energy of the cosmic medium leads to certain form for the additional terms describing the cosmological repulsive forces. Their existence is related to the scale-factor dependence of the thermal energy of the cosmic medium. Analysis of the properties of these forces shows that they are centrifugal in nature. A clear specific example was considered, clarifying their physical meaning. It was shown that the inclusion of terms describing repulsive forces on the right-hand side of the Friedmann cosmological equations does not violate the conservation laws contained in the Einstein equations or their covariance. These terms likewise do not change the equations describing the relationship of the scale factor $a(t)$ with the parameters describing the thermodynamic properties of the cosmic medium.

In the present paper, we consider fact that the generalized Friedmann equations admit a description of the cosmological repulsive forces such that these equations acquire a maximally simple form. This occurs if we suppose that the cosmological repulsive forces precisely balance the effect of the cosmological gravitational forces. In this case, the comoving cosmological reference frame turns out to be inertial. In this frame, the cosmological equations describing the evolution of a uniformly expanding Universe contain only one parameter—the expansion rate. We will denote the model of a uniformly expanding Universe, taking into account its simplicity, the S (simple) model. Unlike the ΛCDM model, the S model does not contain singularities in the behavior of $\dot{a}(t)$ and $\ddot{a}(t)$ as $a(t) \to 0$ and $a(t) \to \infty$.

We do not believe that the S model fully correctly describes the evolution of the Universe. However, as we show in this paper, the S model is apparently a good approximate description of the Universe's dynamics. We believe that this is connected with the following circumstance. The Universe's expansion rate is so high that the influence of the cosmological attraction and repulsive forces, which rapidly decrease as the scale factor $a(t)$ grows, has long been unable ti alter it appreciably. Therefore, the expansion of the Universe is almost uniform, except for a comparatively short initial period.

In the present paper, the S model is used to explain observations important for cosmology corresponding to a wide range of redshifts z (up to $z \sim 1000$). We use this model to interpret the apparent magnitude–redshift relation for Type Ia supernovae, explain the observed angular separation between neighboring bright spots in the uniform cosmic microwave background (CMB), and find the Universe's lifetime. We present an explanation of the above observational data in the ΛCDM model in parallel, in order to better understand which of the models—ΛCDM or the S model—better conforms to the observations.

Below, we present the necessary information used to obtain the equations describing the dynamics of a homogeneous, isotropic Universe taking into account the cosmological repulsive forces. We describe the methodology used to derive these equations.

2. THE EINSTEIN EQUATIONS

Cosmology is underpinned by general relativity (GR). According to this theory, the four-dimensional space–time is non-Euclidean in the presence of matter. The metric properties of space–time are described by the metric

$$ds^2 = g_{ik} dx^i dx^k. \qquad (7)$$

Here and below, the indices $i, j, k, \ldots$ take on the values 0, 1, 2, 3, and the indices α, β, γ the values 1, 2, 3. The metric coefficients g_{ik} are functions of the four space–time coordinates $x_i = (x_0, x_1, x_2, x_3)$. They are connected in a one-to-one fashion with the distribution of matter and the motion of its constituent particles. The properties of matter are described by the energy–momentum tensor T_{ik}. The relationship between the components of the metric tensor g_{ik} and the energy–momentum tensor T_{ik} is determined by the Einstein equations

$$R_i^k - \frac{1}{2}\delta_i^k R = \frac{8\pi G}{c^4} T_i^k, \qquad (8)$$

where R_i^k is the Ricci tensor, R is its trace, and δ_i^k is the Kronecker delta function.

The Ricci tensor has the form

$$R_i^k = g^{ks} R_{is} \qquad (9)$$
$$= g^{ks}\left(\frac{\partial \Gamma_{is}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^s} + \Gamma_{is}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{sm}^l\right).$$

The Christoffel symbols Γ_{ik}^l are defined by the formula

$$\Gamma_{ik}^l = g^{lm} \Gamma_{m,ik} \qquad (10)$$
$$= \frac{1}{2} g^{lm}\left(\frac{\partial g_{mi}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^m}\right).$$

In cosmology, the cosmic medium is usually described as a continuous, perfect fluid, with the energy–momentum tensor having the form

$$T_i^k = (\varepsilon + P) u_i u^k - P \delta_i^k, \qquad (11)$$

where u_i is the four-velocity of macroscopic motion of the medium.

For more details on the Einstein equations, see e.g., [1, 2, 7, 22, 23].

3. THE GEOMETRY OF A HOMONENEOUS, ISOTROPIC UNIVERSE

To describe the geometry of the homogeneous, isotropic, non-stationary, three-dimensional space of the Universe, it is convenient to use a geometric analogy, considering this space to be a homogeneous and isotropic three-dimensional hypersurface in a fictitious four-dimensional Euclidean space [23, Section 107]. In this space, we can introduce four-dimensional Cartesian, spherical and other coordinate systems in the standard way.

The equation describing a non-stationary, homogeneous and isotropic, three-dimensional hypersurface in the four-dimensional Cartesian coordinates (x_1, x_2, x_3, x_4) has the form

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = ka^2(t). \quad (12)$$

The constant k can take three values: $k = +1, -1$, 0. The value $k = +1$ realizes the case of a space with constant positive curvature. The value $k = -1$ corresponds to a space with negative curvature. Flat space, which has zero curvature, corresponds to $k = 0$. The point $O = (0, 0, 0, 0)$ is the center of the Universe, and $\sqrt{k}a(t)$ is its radius. The radius of a non-stationary Universe a changes in time. Let us consider the geometric properties of spaces with $k = +1, -1, 0$ separately.

3.1. A Spherical Universe ($k = +1$)

When $k = +1$, the space of a homogeneous and isotropic Universe is a three-dimensional hypersphere, which it is convenient to describing using the four-dimensional spherical coordinate system (a, χ, θ, φ). In this system, the center of the Universe is the point where $a = 0$. The four-dimensional Cartesian and spherical coordinates are related by the expressions

$$x_1 = a \sin\chi \sin\theta \cos\phi, \quad (13)$$
$$x_2 = a \sin\chi \sin\theta \sin\phi,$$
$$x_3 = a \sin\chi \cos\theta, \quad x_4 = a \cos\chi.$$

The admissible ranges of the spherical coordinates are

$$0 \le a \le \infty, \quad 0 \le \chi \le \pi, \quad (14)$$
$$0 \le \theta \le \pi, \quad 0 \le \varphi \le 2\pi.$$

To describe a spherical Universe, we will also use a three-dimensional curvilinear coordinate system, namely, the comoving system. We will also call this the coordinate system of typical observers. A typical observer is an abstract object of the Universe that undergoes only radial motion relative to its center in the four-dimensional spherical coordinate system. The system of typical observers is an infinite set that fills the Universe in a homogeneous manner.

We will choose the temporal coordinate such that, in the comoving coordinate system of any typical observer, the interval between two infinitely close events occurring at the point where he is located is given by the relation

$$ds^2 = c^2dt^2. \quad (15)$$

Since all typical observers are equivalent, the time introduced in this way is the same for all observers and is therefore called the world time.

We will study the dynamics of the Universe with respect to one typical observer, calling him the "main" observer. Due to the equivalence of all typical observers, any of them could be the "main" observer. Let us choose the four-dimensional spherical coordinate system such that the "main" typical observer ($\bar{M}$) is located at the "North" pole of this coordinate system.

To describe the motion of an arbitrary typical observer M relative to the main observer in the comoving reference frame $\bar{\mathrm{M}}$, it is convenient to use a comoving three-dimensional coordinate system. By definition, in this system, the coordinates of the point M are χ, θ, φ, so that the distance from $\bar{\mathrm{M}}$ to M is $R(t) = a(t)\chi$. The scale factor $a(t)$ describes homogeneous and isotropic extension and compression of the comoving coordinate system.

We will assume that the homogeneity and isotropy of space are preserved during the Universe's evolution. Under this condition, any typical observer M moves only along a radial coordinate line of the four-dimensional spherical coordinate system. His equations of motion in this system are

$$a = a(\tau), \quad \chi(\tau) = \chi_0, \quad (16)$$
$$\theta(\tau) = \theta_0, \quad \varphi(\tau) = \varphi_0,$$

where

$$a(\tau_0) = a_0, \quad \chi(\tau_0) = \chi_0, \quad (17)$$
$$\theta(\tau_0) = \theta_0, \quad \varphi(\tau_0) = \varphi_0$$

are the spherical coordinates of M at the initial time $\tau = \tau_0$. The quantity τ determines time in the four-dimensional spherical coordinate system.

In the comoving system, the motion of M relative to the main typical observer $\bar{M}$ is given by the equations

$$R(t) = a(t)\chi_0, \quad \chi(t) = \chi_0, \quad (18)$$
$$\theta(t) = \theta_0, \quad \varphi(t) = \varphi_0.$$

We see that the motion of M relative to $\bar{\mathrm{M}}$ is subject to the Hubble law

$$dR(t)/dt = H(t)R(t), \quad (19)$$

where

$$H(t) = (da/dt)/a \quad (20)$$

is the Hubble parameter, which is the same for any $R(t)$.

The Hubble law is an "internal" law determining the motion of an arbitrary typical observer relative to the main observer. It is a consequence of the preservation of the Universe's homogeneity and isotropy during its evolution.

The spherical space is non-Euclidean. If (χ, θ, φ) and $(\chi + d\chi, \theta + d\theta, \varphi + d\varphi)$ are the coordinates of two infinitely close points in the comoving coordinate system, then the squared spatial distance between them is

$$dl^2 = a^2\Big\{d\chi^2 + \sin^2\chi\left[\sin^2\theta(d\varphi)^2 + (d\theta)^2\right]\Big\}. \quad (21)$$

The interval between two infinitely close events in the comoving reference frame is written, with (15) and (21), as

$$ds^2 = c^2dt - a^2(t) \times \left\{d\chi^2 + \sin^2\chi\left[\sin^2\theta(d\varphi)^2 + (d\theta)^2\right]\right\}. \quad (22)$$

The circumference of the radius $R = a\chi$ is $L = 2\pi a \sin\chi$. The ratio of the circumference to the radius is $L/R = 2\pi\,(\sin\chi/\chi) < 2\pi$. The area of a sphere of radius $R = a\chi$ is equal to

$$S(R) = a^2\sin^2\chi\int\limits_0^{2\pi}d\varphi\int\limits_0^{\pi}\sin\theta d\theta = 4\pi a^2\sin^2\chi. \quad (23)$$

The radius R of the sphere can vary in the range $0 \le R \le \pi a$. When $R = 0$, we have $S(0) = 0$. At first, as R grows, the quantity $S(R)$ also grows, reaching its maximum, $S_{\max} = 4a^2\pi$, at $R = \pi a/2$. Upon further growth of R, $S(R)$ decreases, vanishing at $R = \pi a$.

The spatial volume of a Universe whose radius of curvature at any point is a is given by the relation

$$V = a^3\int\limits_0^{2\pi}d\varphi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^{\pi}\sin^2\chi d\chi = \int\limits_0^{\pi}S(R)ad\chi = 2\pi^2a^3. \quad (24)$$

3.2. Space of Negative Curvature ($k = -1$)

The relations describing the geometry of a homogeneous space of negative curvature are obtained from those describing a spherical Universe if we formally substitute $a \to ia$, $\chi \to i\chi$. With $k = -1$, Eq. (12) describes a three-dimensional pseudosphere.

It is convenient to describe this pseudosphere using the coordinates $(a, \chi, \theta, \varphi)$, which are related to the Cartesian coordinates as

$$x_1 = a\sinh\chi\sin\theta\cos\varphi, \quad x_2 = a\sinh\chi\sin\theta\sin\varphi, \quad (25)$$
$$x_3 = a\sinh\chi\cos\theta, \quad x_4 = -ia\cosh\chi.$$

Here, the admissible ranges of the pseudospherical coordinates are

$$0 \le a \le \infty, \quad 0 \le \chi < \infty, \quad 0 \le \theta \le \pi, \quad 0 \le \varphi \le 2\pi. \quad (26)$$

The pseudospherical space of the Universe is non-Euclidean. If (χ, θ, φ) and $(\chi + d\chi, \theta + d\theta, \varphi + d\varphi)$ are the coordinates of two infinitely close points in the comoving coordinate system, the squared spatial distance between them is

$$dl^2 = a^2\Big\{d\chi^2 + \sinh^2\chi\left[\sin^2\theta(d\varphi)^2 + (d\theta)^2\right]\Big\}. \quad (27)$$

The interval between two infinitely close events is written in the comoving coordinate system, with (15) and (27), as

$$ds^2 = c^2dt^2 - a^2(t)\Big\{d\chi^2 + \sinh^2\chi\left[\sin^2\theta(d\varphi)^2 + (d\theta)^2\right]\Big\}. \quad (28)$$

The radius of a circle on the pseudosphere is $R = a\chi$. The circumference of the radius R is $L = 2\pi a\sinh\chi$. The circumference-to-radius ratio $L/R = 2\pi a\,(\sinh\chi/\chi)$ is greater than 2π. The area of a sphere of radius $R = a\chi$ is equal to

$$S(R) = 4\pi a^2\sinh^2\chi. \quad (29)$$

The spatial volume encompassed by a pseudosphere of radius $R = a\chi$ is

$$V(R) = \int\limits_0^{2\pi}d\varphi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^{\chi}a^3\sinh^2\chi d\chi = \pi a^2\left(\sinh^2 2\chi - 2\chi\right). \quad (30)$$

As the radius $R = a\chi$ grows, both $S(R)$ and $V(R)$ continuously grow.

3.3. Flat (Euclidean) Space ($k = 0$)

The case of an infinite radius of curvature of the three-dimensional space is a limiting one, in which the space of the Universe is flat. The interval ds^2 can be written in this case

$$ds^2 = c^2dt^2 - b^2(t)\left(dx^2 + dy^2 + dz^2\right). \quad (31)$$

It is convenient to use the Cartesian coordinates x, y, z as the spatial coordinates. The time-dependent factor $b^2(t)$ in the relation determining the squared element of length,

$$dl^2 = b^2(t)\left(dx^2 + dy^2 + dz^2\right), \qquad (32)$$

in the flat Universe does not alter the Euclidean nature of the spatial metric. At any given t, the factor $b^2(t)$ has some value, and can be made equal to unity via a simple coordinate transformation.

During the evolution of a flat Universe, the "frozen-in" Cartesian coordinate system suffers a homogeneous deformation. In this space, a circumference of radius a is $2\pi a$, and the area of a sphere is $4\pi a^2$. The volume encompassed by a sphere of radius a is

$$V = \frac{4}{3}\pi a^3. \qquad (33)$$

Formally, the flat space may be described mathematically in spherical coordinates, so that ds^2 is written in the form

$$ds^2 = c^2dt^2 - a^2(t)\Big\{d\chi^2 + \chi^2\left[\sin^2\theta(d\varphi)^2 + (d\theta)^2\right]\Big\}. \qquad (34)$$

4. THE FRIEDMANN COSMOLOGICAL EQUATIONS

The metric of a homogeneous, isotropic space–time contains only one scalar parameter, namely, the scale factor a. This determines the curvature of space. The Einstein equations for a homogeneous, isotropic Universe can be transformed to Friedmann's cosmological equations, which determine the relationship between the scale factor a and the quantities describing the thermodynamic properties of the cosmic medium (see, e.g., [1, 2]). These equations were first obtained and used to describe the Universe by Friedmann [24, 25], and form the basis for our studies.

The medium is assumed to be perfect, with the energy–momentum tensor T_i^k determined by (11). Using (11) to calculate the energy–momentum tensor, we neglect all dissipation processes that lead to entropy growth. On the validity of this approach see, e.g., [23, Section 108].

When obtaining Friedmann's equations, we use the comoving coordinate system, with respect to which the medium is at rest, so that the four-velocity components are $u^i = (1, 0, 0, 0)$; then, only the following components T_i^k turn out to be nonzero:

$$T_0^0 = \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = -P. \qquad (35)$$

To describe the geometry of a homogeneous, isotropic three-dimensional hypersurface, the Friedmann equations use the three-dimensional curvilinear coordinate system, i.e., the comoving system (see Section 3). Let us present a brief derivation of Friedmann's cosmological equations from the Einstein equations for the case of a closed homogeneous, isotropic spherical space. For such a space, the interval between two infinitely close events in the comoving coordinate system is written in the form [see (22)]

$$ds^2 = c^2dt^2 - a^2(t)\Big\{d\chi^2 + \sin^2\chi[\sin^2\theta(d\varphi)^2 + (d\theta)^2]\Big\}. \qquad (36)$$

Instead of the variable t, we will use the variable η defined by the relation

$$cdt = ad\eta, \qquad (37)$$

where η is a dimensionless time variable; $\eta = x^0$. In this case, the interval ds^2 determined by (36) is written in the form

$$ds^2 = a^2(\eta)\Big\{(d\eta)^2 - (d\chi)^2 - \sin^2\chi\left[(d\theta)^2 + \sin^2\theta(d\varphi)^2\right]\Big\}. \qquad (38)$$

Assuming $x^0 = \eta$, $x^1 = \chi$, $x^2 = \theta$, $x^3 = \varphi$ and writing ds^2 in the standard form (7), we find the components of the metric tensor g_{ik}:

$$g_{00} = a^2, \quad g_{11} = -a^2, \quad g_{22} = -a^2\sin^2\chi, \qquad (39)$$
$$g_{0\alpha} = 0, \quad g_{\alpha\beta} = 0, \quad g_{33} = -a^2\sin^2\chi\sin^2\theta.$$

The determinant $g = \det(g_{ik})$ is equal to

$$g = -a^8\sin^4\chi\sin^2\theta. \qquad (40)$$

The components of the contravariant metric tensor g^{ik} are, by definition,

$$g^{ik} = G_{(ik)}/g, \qquad (41)$$

where $G_{(ik)}$ are the minors corresponding to the elements g_{ik} in the determinant $\det(g_{ik})$. It follows from the theory of determinants that

$$g_{ik}g^{km} = \delta_i^m. \qquad (42)$$

Using (40), (41), and (42), we find:

$$g^{00} = a^{-2}, \quad g^{11} = -a^{-2}, \qquad (43)$$
$$g^{22} = -a^{-2}\sin^{-2}\chi, \quad g^{33} = -a^{-2}\sin^{-2}\chi\sin^{-2}\theta,$$
$$g^{0\alpha} = 0, \quad g^{\alpha\beta} = 0.$$

Calculating the components of the Ricci tensor (9), we obtain:

$$R_0^0 = \frac{3}{a^4}\left[\left(a'\right)^2 - aa''\right], \qquad (44)$$

$$R_\alpha^0 = 0, \quad R_\alpha^\beta = -\frac{1}{a^4}\left[2a^2 + (a')^2 + aa''\right]\delta_\alpha^\beta.$$

Here, a prime denotes a derivative with respect to η.

The method used to calculate the Ricci tensor components R_k^i based on the symmetry of the Riemann tensor in the homogeneous, isotropic Universe is presented in [23, Section 107]. Using (44), we find the trace of the Ricci tensor:

$$R = R_0^0 + R_\alpha^\alpha = -\frac{6}{a^3}\left(a + a''\right). \qquad (45)$$

It follows from (44) and (45) that

$$R_0^0 - \frac{1}{2}R = 3\left[\left(\frac{a'}{a^2}\right)^2 + \frac{1}{a^2}\right], \qquad (46)$$

$$R_1^1 - \frac{1}{2}R = R_2^2 - \frac{1}{2}R = R_3^3 - \frac{1}{2}R \qquad (47)$$
$$= 2\frac{a''}{a^3} - \left(\frac{a'}{a^2}\right)^2 + \frac{1}{a^2}.$$

Taking into account (35), (44)–(47), we conclude that, for a space–time with the metric (38) that is homogeneously filled with a perfect medium, the Einstein equations reduce to the two equations

$$3\left[\left(\frac{a'}{a^2}\right)^2 + \frac{1}{a^2}\right] = \frac{8\pi G}{c^4}\varepsilon, \qquad (48)$$

$$2\frac{a''}{a^3} - \left(\frac{a'}{a^2}\right)^2 + \frac{1}{a^2} = -\frac{8\pi G}{c^4}P. \qquad (49)$$

Passing from the variable η to the variable t, we obtain from (37)

$$a' = \frac{a}{c}\dot{a}, \quad a'' = \frac{a}{c^2}(a\ddot{a} + \dot{a}^2). \qquad (50)$$

With (50), Eqs. (48) and (49) are brought to the form

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{c}{a}\right)^2\right] = \frac{8\pi G}{c^2}\varepsilon, \qquad (51)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{c}{a}\right)^2 = -\frac{8\pi G}{c^2}P. \qquad (52)$$

Here, a dot denotes a derivative with respect to the time t.

Analogous calculations can be conducted for the cases $k = 0$ and $k = -1$. Their common result for the cases $k = 0, \pm 1$ are the equations

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2}\right] = \frac{8\pi G}{c^2}\varepsilon, \qquad (53)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}P. \qquad (54)$$

These are called the Friedmann cosmological equations. It can easily be shown that they can be transformed to

$$\frac{d\varepsilon}{da} + 3\left(\varepsilon + P\right)\frac{1}{a} = 0, \qquad (55)$$

$$\ddot{a} = -\frac{4}{3}\pi G\frac{a}{c^2}\left(\varepsilon + 3P\right). \qquad (56)$$

We can see from (56) that the contribution of the pressure (thermal energy) to the creation of a cosmological gravitational acceleration can be significant. This occurs in cases when the pressure is comparable to the energy density of the cosmic medium. In the cosmic medium, $P > 0$, and, according to (56), the effect of the pressure is to decelerate rather than to accelerate the Universe's expansion rate.

The viewpoint that pressure can only decelerate the cosmological expansion is conventional (see, e.g., [1, Ch. 1]). The suggestion that pressure (thermal energy) can influence the direction of the expansion rate of a homogeneous cosmic medium is perceived negatively. Everybody understands that there are no pressure gradients in a homogeneous medium, and therefore there are no outward-pushing forces due to the pressure. According to the standard form of Friedmann's equations, the thermal energy of a homogeneous, isotropic medium not only cannot change the sign of the cosmological acceleration, but, as can be seen from (56), it can only enhance the effect of gravity. Note that this conclusion is obtained from the standard Friedmann equations, which do not take into account the effect of cosmological repulsive forces on the cosmic medium. It was shown in [21] that a significant role can be played by centrifugal cosmological repulsive forces in a homogeneous and isotropic cosmic medium. These are connected with thermal-energy changes in the cosmic medium in curved space. Their effect can lead to an increase in the expansion rate of the cosmic medium.

5. EINSTEIN EQUATIONS WITH Λ COSMOLOGICAL CONSTANT

The Einstein equations (8) do not contain repulsive forces. A version of the GR equations that contains repulsive forces was put forward by Einstein [26], via the introduction of the so-called Λ term containing the cosmological constant Λ, bringing the Einstein equations into the form

$$R_i^k - \frac{1}{2}\delta_i^k R = \frac{8\pi G}{c^4}T_i^k + \delta_i^k \Lambda. \qquad (57)$$

The value of the universal constant Λ can be found by comparing theoretical predictions with observations. It is supposed that $\Lambda \approx 10^{-56}$ cm^{-2} [1, 6, 7].

Taking into account the Einstein repulsive forces described by the Λ term in the Einstein equations leads to the appearance of additional terms on the right-hand sides of the Friedmann equations (53) and (54), which take the form

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2}\right] = \frac{8\pi G}{c^2}\varepsilon_{\text{eff}} \quad (58)$$
$$= \frac{8\pi G}{c^2}\varepsilon + c^2\Lambda,$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}P_{\text{eff}} \quad (59)$$
$$= -\frac{8\pi G}{c^2}P + c^2\Lambda,$$

(for details, see, e.g., [1, Chapter 4]).

The transition from (8) to (57) means the substitution

$$T_i^k \Rightarrow T_{i\text{eff}}^k = (\varepsilon_{\text{eff}} + P_{\text{eff}})\, u_i u^k - P_{\text{eff}}\delta_i^k, \quad (60)$$

$$\varepsilon \Rightarrow \varepsilon_{\text{eff}} = \varepsilon + \varepsilon_\Lambda, \quad P \Rightarrow P_{\text{eff}} = P + P_\Lambda, \quad (61)$$

where

$$\varepsilon_\Lambda = c^4\Lambda/8\pi G, \quad P_\Lambda = -\varepsilon_\Lambda. \quad (62)$$

In our view, it is important to understand that ε_Λ and P_Λ are not corrections to ε and P, but instead independent quantities that are sources of repulsive forces, which are fundamentally different from gravitational forces.

Equations (58) and (59) can straightforwardly be used to obtain a relation determining the cosmological acceleration due to the Einsteinian repulsive forces, which has the form [1, Chapter 4]

$$\ddot{a}_\Lambda = -\frac{4}{3}\pi G\frac{a}{c^2}\left(\varepsilon_\Lambda + 3P_\Lambda\right) = \frac{1}{3}\Lambda c^2 a. \quad (63)$$

For the Einsteinian repulsive forces, whose source are the quantities ε_Λ and P_Λ, it is significant that $P_\Lambda = -\varepsilon_\Lambda$; precisely this is the reason for the repulsive nature of these forces. At the same time, even when $G \equiv 0$, there remains the Einsteinian repulsion force field (non-removable curvature), and we therefore consider at to be an entity independent of the gravitational field.

6. GENERALIZED EINSTEIN EQUATIONS

We have noted that, apart from Einsteinian forces related to the cosmological constant, other cosmological repulsive forces are theoretically possible in the Einstein equations. The standard Einstein equations (8) admit a substitution of a more general nature than (60) and (61). This has the form

$$T_i^k \Rightarrow T_{i,\text{eff}}^k, \quad \varepsilon \Rightarrow \varepsilon_{\text{eff}} = \varepsilon + \varepsilon_\Delta, \quad (64)$$

$$P \Rightarrow P_{\text{eff}} = P + P_\Delta.$$

The quantities ε_Δ and P_Δ are defined by the relations

$$\varepsilon_\Delta = -\frac{3c^2}{8\pi G}\frac{\Delta^2(a)}{a^2}, \quad (65)$$
$$P_\Delta = \frac{c^2}{8\pi G}\left(\frac{\Delta^2(a)}{a^2} + \frac{1}{a}\frac{d\Delta^2(a)}{da}\right),$$

where $\Delta^2(a)$ is an arbitrary function of the scale factor a. The quantity a is a scalar characterizing the properties of a homogeneous and isotropic Universe, which vary in the same way at all its points. The quantities ε_Δ and P_Δ are a source of cosmological repulsive forces. Like ε and P, they are scalar functions. The Einsteinian sources of repulsive forces ε_Λ and P_Λ are a special case of the quantities ε_Δ and P_Δ introduced here.

With (64), the Einstein equations acquire the form

$$R_i^k - \frac{1}{2}\delta_i^k R = \frac{8\pi G}{c^4}T_{i,\text{eff}}^k. \quad (66)$$

These equations describe not only the gravitational field but also a cosmological field of repulsive forces whose sources are the quantities ε_Δ and P_Δ. Let us again stress that the existence of the terms ε_Δ and P_Δ in the suggested expressions for ε_{eff} and P_{eff} does not at all mean that these are corrections to the energy and pressure. The quantities ε_{eff} and P_{eff} consist of two parts. The first of these, ε and P, are sources of the gravitational field, while ε_Δ and P_Δ are sources of cosmological repulsive forces. Our suggested repulsive forces, like the Einsteinian ones, are independent of the gravitational field. To distinguish (66) from the standard Einstein equations (8), we will call the former the generalized Einstein equations.

In the transformations (64) and (65), the matrix $T_{i\text{eff};k}^k$ remains a second-rank tensor, so that the covariance of the Einstein equations is not violated. The substitution (64), (65) likewise does not violate the conservation laws $T_{i\,\text{eff};\,k}^k = 0$ contained in the Einstein equations. Under any choice of the function $\Delta^2(a)$, the conservation laws are valid:

$$T_{i\,\text{eff};\,k}^k = 0. \quad (67)$$

With any form of the function $\Delta^2(a)$, the following relation holds:

$$T_{i\,\text{eff};\,k}^k = 0 \Rightarrow T_{i,k}^k = 0. \quad (68)$$

The transformations (60)–(62), leading to the Einstein equations with the cosmological constant, are special cases of the transformations (64), (65). Indeed, if $\Delta^2(a)$ is chosen in the form

$$\Delta^2(a)/2 = \Delta_\Lambda^2(a)/2 = -\frac{1}{6}\Lambda c^2 a^2, \quad (69)$$

then $\varepsilon_\Delta = \varepsilon_\Lambda$, $P_\Delta = P_\Lambda$, and we obtain the Einstein equations with the cosmological constant.

7. GENERALIZED FRIEDMANN EQUATIONS

In the case of a homogeneous, isotropic Universe, the corresponding generalized Friedmann equations are obtained from the generalized Einstein equations in the standard way. The resulting equations are written

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) = \frac{8\pi G}{c^2}\varepsilon - 3\frac{\Delta^2(a)}{a^2}, \quad (70)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \quad (71)$$
$$= -\frac{8\pi G}{c^2}p - \frac{1}{a}\frac{d\Delta^2(a)}{da} - \frac{\Delta^2(a)}{a^2}.$$

These were termed in [21] the generalized Friedmann equations. We consider the Universe to be a homogeneous and isotropic hypersurface in four-dimensional space. We suppose that the generalized Friedmann equations describe a motion of this hypersurface in the fourth spatial dimension, which is perpendicular to it at each point. This dimension is large-scale and is not curved.

In the generalized Friedmann equations (70) and (71), terms $\sim\dot{a}^2$ describe the kinetic energy of the cosmological medium. The additional terms $\sim\Delta^2(a)$ describe the effect of some energy that is a source of cosmological repulsive forces. It is supposed that the description of the kinetic energy of the cosmic medium and the energy that is the source of repulsive forces in these equations should be symmetric. It is this kind of generalization of the Friedmann equations that was carried out in [21]. Formally, this implies the substitution

$$\dot{a}^2 \Rightarrow \dot{a}^2 + \Delta^2(a), \quad (72)$$
$$\ddot{a} \Rightarrow \ddot{a} + \frac{1}{2}\frac{d\Delta^2(a)}{da}.$$

This substitution leads to equations which can describe both attractive and repulsive forces in a homogeneous and isotropic Universe. Substituting (72) into (53) and (54) and using the notation (65), we conclude that the substitutions (72) and (64), (65) are equivalent.

The generalized Friedmann equations (70), (71) can be brought to the form

$$\frac{d\varepsilon}{da} + 3\left(\varepsilon + P\right)\frac{1}{a} = 0, \quad (73)$$

$$\ddot{a} = -\frac{4}{3}\pi G\frac{a}{c^2}\left(\varepsilon + 3P\right) - \frac{d}{da}\left(\frac{\Delta^2(a)}{2}\right). \quad (74)$$

We can easily verify that the first of these equations is the zeroth component of the conservation law (67) for a homogeneous isotropic Universe. The other components of (67) are identically equal to zero for any choice of the function $\Delta^2(a)$.

Note that (73) is the first law of thermodynamics written per unit mass in a homogeneous isotropic Universe. Bearing in mind that the expansion of the Universe is an adiabatic process (see, e.g., [1, 2]), the first law of thermodynamics can be written in the form

$$dE = d\left(\varepsilon V\right) = -PdV. \quad (75)$$

Since, in the case under consideration, $V \sim a^3$, we conclude that (73) follows from (75).

We can see from (74) that using the transformations (64), (65) is actually a method of describing the effect of some energy $\Delta^2(a)/2$ on the Universe's dynamics.

The system of equations (73), (74) is not complete. To close it, we must take into account equations describing the thermodynamic properties of the cosmic medium. It is difficult to write such equations in the general case. During the Universe's evolution, the component composition of the cosmic medium changes, as well as the conditions for the interaction between the components (see, e.g., [1, 2, 7]).

The following two limiting cases are frequently considered in the theory.

7.1. A Non-relativistic Universe

Assuming $P \equiv 0$ and $\varepsilon = \rho c^2$, we find from (73) that, for any k and $\Delta^2(a)$,

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{a}\frac{da}{dt} = 0. \quad (76)$$

It follows that the density of the cosmic medium $\rho(t)$ and the scale factor of the Universe $a(t)$ are related by

$$\rho a^3 = \text{const}. \quad (77)$$

In modern cosmology it is supposed that (73) and (74) with $P = 0$ describe the Universe's dynamics when the contribution of the relativistic component of the cosmic medium to its total mass (energy) is negligibly small.

7.2. A Relativistic Universe

Assuming $P = (1/3)\rho c^2$, we find from (73) that, for any k and $\Delta^2(a)$,

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{4}{a}\frac{da}{dt} = 0. \quad (78)$$

It follows that, in a relativistic Universe, the density, specific energy, and pressure are related to the scale factor as

$$\rho a^4 \sim \varepsilon a^4 \sim P a^4 = \text{const}. \qquad (79)$$

In modern cosmology, it is supposed that (73) and (74) with $P = (1/3)\rho c^2$ describe well the early Universe at the epoch when the contribution of the relativistic component of the cosmic medium to its total mass (energy) was dominant.

8. MODEL OF A UNIFORMLY EXPANDING UNIVERSE

Let us describe the model for a uniformly expanding Universe (S model) we propose here. We will also write equations describing the cosmological model that is the most widespread at present, namely, the ΛCDM model. We will use these equations to compare the calculated characteristics of observational dependences obtained in the S and ΛCDM models.

Let us take into account the multi-component nature of the medium that fills the Universe. We will describe it using a two-component approximation. We will assume that the medium consists of two homogeneously mixed components: (a) non-relativistic and (b) relativistic.

The non-relativistic component includes all elements of the cosmic medium, both observable ("baryonic") and unobservable ("dark matter"), containing particles whose rest mass is much higher than their kinetic energy. The dark matter may be clustered, and is currently the dominating (by mass) part of the non-relativistic component of the cosmic medium. We assume that the effect of the pressure of the non-relativistic component on the Universe's dynamics is insignificant.

The relativistic component will include all elements of the cosmic medium, both observable (the CMB) and unobservable, whose equation of state is $P = (1/3)\varepsilon$. This component consists of particles whose rest mass is either zero or much smaller than their total energy. We assume that the relativistic component is not clustered. Its contribution to the total mass of the cosmic medium is at present small (see, e.g., [6, 7]).

The ratio of the number densities of the non-relativistic (n_1) and relativistic (n_2) components remains constant, except for the earliest stages in the Universe's evolution. We take into account that, according to the observational data, $n_2/n_1 \sim 10^9$, but it is possible that this ratio is much greater.

Note that the description of some constituents of the cosmic medium (such as neutrinos) must be more detailed, and must allow for the possibly finite value of their rest mass.

Our description of the Universe's dynamics using the model suggested begins with the generalized Friedmann equations (70), (71). For our two-component cosmic medium, these equations are written in the form

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) = 8\pi G(\rho_1 + \rho_2) - \frac{3\Delta^2}{a^2}, \qquad (80)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) = -\frac{8}{3}\pi G\rho_2 - \frac{\Delta^2}{a^2} - \frac{1}{a}\frac{d\Delta^2}{da}. \qquad (81)$$

Here and below, the subscripts "1" and "2" are used to denote the non-relativistic and relativistic components, respectively.

When writing (80) and (81), we assumed that the full pressure of the cosmic medium is $P = P_1 + P_2 \approx P_2 = (1/3)\varepsilon_2$. The energy densities ε_1 and ε_2 are related to the densities ρ_1 and ρ_2 by the expressions $\varepsilon_1 = \rho_1 c^2$, $\varepsilon_2 = \rho_2 c^2$.

Equations (80), (81) can easily be transformed to

$$\ddot{a} = -\frac{4}{3}\pi G a(\rho_1 + 2\rho_2) - \frac{1}{2}\frac{d\Delta^2}{da}, \qquad (82)$$

$$\frac{d}{da}(\varepsilon_1 + \varepsilon_2) + (3\varepsilon_1 + 4\varepsilon_2)\frac{1}{a} = 0. \qquad (83)$$

When $\Delta^2(a) \neq \text{const}$, Eq. (82) describes not only the action of gravitational forces, but also of repulsive forces. Equation (83) describes the energy change of the two-component cosmic medium during the Universe's evolution. It has this form for any choice of $\Delta^2(a)$ and any value of the parameter k.

In the two-component model of the cosmic medium, (83) divides into two equations:

$$\frac{d\rho_1}{da} + 3\rho_1\frac{1}{a} = 0, \qquad (84)$$

$$\frac{d\rho_2}{da} + 3\rho_2\frac{1}{a} = 0. \qquad (85)$$

Integrating these equations, we conclude that the densities of the non-relativistic (ρ_1) and relativistic (ρ_2) components are related to the characteristic size of the Universe $a(t)$ as

$$\rho_1(a) = \rho_{10}(a_0/a)^3, \qquad \rho_2(a) = \rho_{20}(a_0/a)^4. \qquad (86)$$

With (86), Eqs. (80), (81) can be written

$$\frac{\dot{a}^2}{2} + \frac{\Delta^2}{2} - \frac{\tau_1}{a} - \frac{\tau_2}{2a^2} = -\frac{kc^2}{2}, \qquad (87)$$

$$\ddot{a} = -\frac{d}{da}\left(-\frac{\tau_1}{a} - \frac{\tau_2}{2a^2}\right) - \frac{1}{2}\frac{d\Delta^2}{da}. \qquad (88)$$

The constants τ_1 and τ_2 are defined by the relations

$$\tau_1 = \frac{4}{3}\pi G\rho_{10}a_0^3, \qquad \tau_2 = \frac{8}{3}\pi G\rho_{20}a_0^4. \qquad (89)$$

Equation (87) is interpreted as representing energy conservation per unit total mass of the cosmic medium. According to this equation, the sum of the kinetic energy of the cosmic medium's expansion $\left(\dot{a}^2/2\right)$, the energy $\Delta^2(a)/2$, which is a source of repulsive forces, and the potential energy $-\left(\tau_1/a + \tau_2/2a^2\right)$, which is a source of attractive forces, remains constant during the Universe's evolution.

Equation (88) represents the radial motion of the cosmic medium in the fictitious four-dimensional space. The first term on the right-hand side of (88) describes the action of attractive forces, and is related to changes in the potential energy of the cosmic medium. The second term describes repulsive forces, which are related to changes in the energy $\Delta^2(a)/2$. It is obvious that a necessary condition for the presence of volume repulsive forces in a homogeneous isotropic Universe is that the energy $\Delta^2(a)/2$ be variable during the evolution.

The standard Friedmann equations without a cosmological constant correspond to $\Delta^2(a) \equiv 0$. These equations describe the old classical cosmological model (see, e.g., [1]). The Friedmann equations with a cosmological constant correspond to "dark energy":

$$\frac{1}{2}\Delta^2_\Lambda(a) = -\frac{1}{6}\Lambda c^2 a^2, \tag{90}$$

where Λ is the cosmological constant. With (90), the Friedmann equations (80), (81) take the form (58), (59). These are components of the Einstein equations with a Λ term for a homogeneous and isotropic Universe. They underlie the ΛCDM model [6, 7]. Let us write them in the form

$$\left(\frac{1}{\bar{a}}\frac{d\bar{a}}{d\bar{t}}\right)^2 = \Omega_{\rm curv}(\bar{a})^{-2} + \Omega_M(\bar{a})^{-3} + \Omega_{\rm rad}(\bar{a})^{-4} + \Omega_\Lambda, \tag{91}$$

$$\frac{d^2\bar{a}}{d\bar{t}^2} = -\frac{\Omega_M}{2\bar{a}^2}\left(1 + \frac{2\bar{a}_{eq}}{\bar{a}}\right) + \Omega_\Lambda \bar{a}, \tag{92}$$

where

$$\Omega_\Lambda = \rho_\Lambda/\rho_c, \quad \rho_\Lambda = \Lambda c^2/8\pi G, \quad \bar{a} = a/a_0, \quad \bar{t} = tH_0. \tag{93}$$

When writing (91) and (92) we have used the standard notation [7]

$$\Omega_M = \rho_{10}/\rho_c, \quad \Omega_{\rm rad} = \rho_{20}/\rho_c, \quad \Omega_{\rm curv} = \rho_{{\rm curv},0}/\rho_c, \quad \bar{a}_{eq} = \Omega_{\rm rad}/\Omega_M, \tag{94}$$

where ρ_{10} and ρ_{20} are the modern values of the nonrelativistic and relativistic components, respectively. The quantities ρ_c and $\rho_{{\rm curv},0}$ are, by definition,

$$\rho_c = 3H_0^3/8\pi G, \quad \rho_{{\rm curv},0} = -3kc^2/8\pi G a_0^2. \tag{95}$$

Solutions to (91), (92) must satisfy the initial conditions

$$\bar{a}(\bar{t}_0) = 1, \quad (d\bar{a}/d\bar{t})(\bar{t}_0) = 1, \tag{96}$$

where $\bar{t}_0$ is the present age of the Universe and H_0 is the Hubble constant. We assume that the time $\bar{t} = 0$ corresponds to the Big Bang. Analysis of available observational data gives [27, 28]

$$H_0 = 73 \pm 3\ {\rm km\,s^{-1}Mpc^{-1}}. \tag{97}$$

The Hubble constant H_0 is often written in the form $H_0 = h \cdot 100\ {\rm km\,s^{-1}Mpc^{-1}}$. In calculations, one usually sets $h = 0.7$ [7].

The critical density value ρ_c is defined by the relation

$$\rho_c = 3H_0^2/8\pi G = 1.88 \times 10^{-29} h^2\ {\rm g/cm^3}. \tag{98}$$

The parameters $\Omega_M, \Omega_{\rm rad}, \Omega_\Lambda$ determine the densities of the non-relativistic and relativistic components and dark energy, respectively, in units of ρ_c.

The solutions that describe the Universe's dynamics in the framework of the ΛCDM model contain singularities in the behavior of $a(t)$, determining the time evolution of the characteristic scale of the Universe. As $a(t) \to 0$ and $a(t) \to \infty$, the functions $\dot{a}(t)$ and $\ddot{a}(t)$ grow without bound [see (91) and (92)].

The nature of the cosmological repulsive forces has not been fully established. Therefore, we believe that it is expedient to consider a non-Einsteinian choice of $\Delta^2(a)$, which describes the cosmological repulsive forces in the framework of the generalized Einstein equations. Here, we suggest a form of $\Delta^2(a)$ that removes the above singularities in the behavior of the scale factor $a(t)$, and the generalized Friedmann equations have the simplest possible form. We suggest that Nature may choose this simplest form among all possible ways of evolution. The form of $\Delta^2(a)$ that allows us to achieve these aims is

$$\Delta^2(a) = \frac{2\tau_1}{a} + \frac{\tau_2}{a^2} - \left(k\,c^2 + \gamma^2 c^2\right), \tag{99}$$

where γ is a constant of this model.

If the function $\Delta^2(a)$ is chosen in the form (99), the generalized Friedmann equations (87), (88) take the form

$$\dot{a}^2 = \gamma^2 c^2, \qquad \ddot{a} = 0. \tag{100}$$

According to these equations, the Universe is expanding at a constant rate, equal to γc. The quantity γ is one of the parameters of the uniformly expanding model of the Universe (S model). The value of this parameter should be taken such that the model correctly describes the observations.

We assume that the Universe has emerged as a result of the Big Bang, and the solution to equation (100) satisfies the initial conditions

$$a(0) = 0, \qquad da/dt(0) = \gamma c. \tag{101}$$

According to (100) and (101), the scale factor $a(t)$ changes according to

$$a(t) = \gamma ct. \tag{102}$$

The Hubble law is valid in a homogeneous isotropic Universe with any form of $a(t)$:

$$da/dt = H(t)a(t). \tag{103}$$

The Hubble constant is $H_0 = H(t_0)$, where t_0 is the age of the Universe. With (102) and (103), we find

$$H_0 = (da/dt)\,(t_0)/a(t_0) = 1/t_0. \tag{104}$$

The age of the Universe t_0 and the Hubble constant H_0 in the S model are related by this expression. Since $H_0 = h \cdot 100$ km s^{-1}Mpc^{-1} and $h \approx 0.7$, we find $t_0 \approx 14 \times 10^9$ years. This is believed in modern cosmology to be the lifetime of the Universe (see, e.g., [6, 7]). The equations determining the evolution of the scale factor $a(t)$ in the S model and the ΛCDM model are fundamentally different. The equations that describe the relationship of $a(t)$ and the parameters describing the thermodynamic properties of the cosmic medium [see, e.g., (76)–(79)], do not depend on the choice of $\Delta^2(a)$, and are the same.

The radiation temperature $T(t)$ and the scale factor $a(t)$ are related by

$$T(t)a(t) = T_0 a_0 = \text{const}. \tag{105}$$

Since $t_0 = H_0^{-1}$ and $a(t) = \gamma ct$ in the S model, we find from (105) the time at which the temperature T is achieved:

$$t = t_0 \frac{T_0}{T}. \tag{106}$$

According to modern data (see, e.g., [7]), $T_0 \approx 2.725$ K.

Figure 2 shows a schematic of the most important epochs in the evolution of the Universe and a comparison of their dates in the S model and ΛCDM model. The expansion rate of the early Universe is much lower in the S model than in the ΛCDM model. This difference could be significant in calculating the primordial chemical composition of the cosmic medium. The conditions for cosmological nucleosynthesis are more favorable in the S model than in the ΛCDM model. The duration of the high-temperature epoch is longer in the S model than that in the ΛCDM model by many orders of magnitude.

9. INTERPRETATION OF THE OBSERVED APPARENT MAGNITUDE–REDSHIFT RELATION FOR TYPE IA SUPERNOVAE

One of the effective methods of testing the correctness of a cosmological model is by comparing the apparent magnitude–redshift relation theoretically calculated in the model with the observed relation [1, 3, 4]. These calculations use the formula determining the relation between the visible brightness and redshift for a source whose absolute luminosity is assumed to be known. Let us present a brief derivation of this formula.

9.1. Apparent Magnitude–Redshift Relation

In an expanding Universe, the wavelength λ of a photon emitted at time t and its wavelength λ_0 detected by an observer at time t_0 are related by

$$\lambda_0/\lambda = a_0/a. \tag{107}$$

The quantities a and a_0 determine the characteristic size of the Universe at the times t and t_0, respectively.

The redshift z of an observed object is defined as

$$z = (\lambda_0 - \lambda)/\lambda = a_0/a - 1. \tag{108}$$

The farther the object emitting the photons, the longer their travel time in the expanding Universe, and the greater the ratio $a_0/a(t)$, and hence the redshift z, according to the relation $a_0/a = 1 + z$ (see, e.g., [1]). The redshift z of an object is a directly observed quantity. A measurement of z consists in identifying an emission (or absorption) line or line system and determining how much these lines are shifted toward longer wavelengths. Equations (107) and (108) are general and valid for any z.

In the ΛCDM, it is usually assumed that the Universe is open and has the metric

$$ds^2 = c^2dt^2 - a^2(t)\Big\{(d\chi)^2 + \sinh^2\chi\left[(d\theta)^2 + \sin^2\theta(d\varphi)^2\right]\Big\} \tag{109}$$

(see, e.g., [7]). The reason for this assumption is as follows. In the absence of cosmological repulsive forces, a closed model of the Universe can be realized when $\Omega_M + \Omega_{\rm rad} > 1$. This holds if the density of the cosmic medium is higher than the critical density (see, e.g., [1, 2]). Taking into account repulsive forces, the value of $\Omega_M + \Omega_{\rm rad}$ for which the Universe can be closed should be higher than in the absence of such forces, i.e., greater than unity.

The parameter Ω_M contains contributions from two constituents: "baryonic matter" and "dark matter". Observational estimates of the baryonic component Ω_M are approximately 0.04–0.05 (see, e.g., [6, 7]). On the other hand, numerous observational

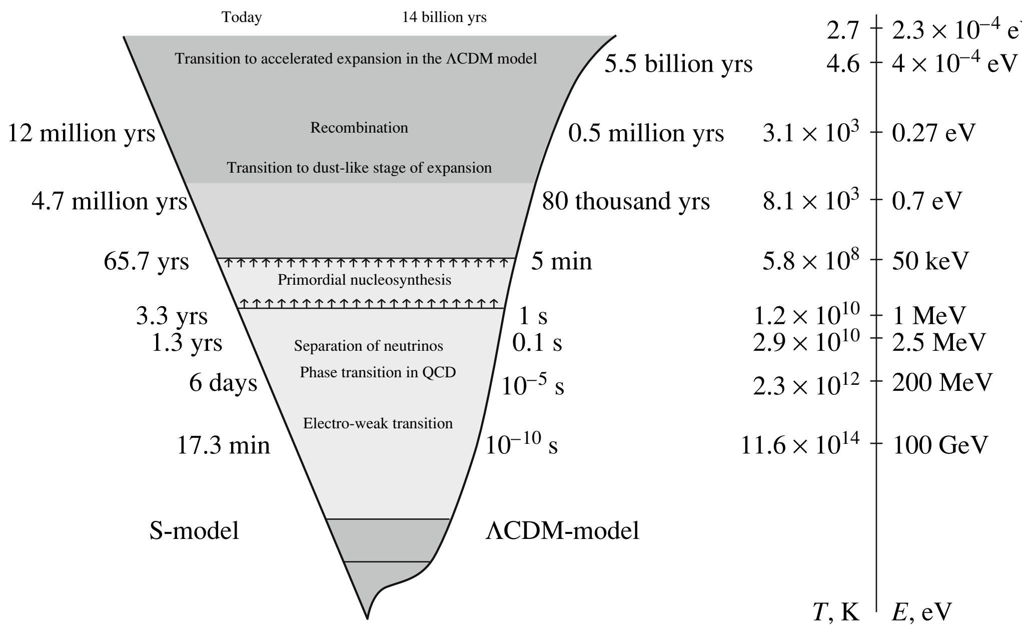

Fig. 2. Schematic of the most important epochs in the evolution of the Universe and comparison of their dates in the S model and ΛCDM model.

data (see, e.g., [17–19]) imply that the amount of dark matter exceeds the amount of baryonic matter by a factor of five to six. Thus, taking into account the dark-matter contribution, it is supposed that the value of Ω_M lies in the range 0.25–0.30 [16–19]. In modern cosmology, the contribution of the relativistic component to the total energy density of the modern Universe is believed to be very small. It is thought that the value of $\Omega_{\rm rad}$ is $\approx \left(4.2/h^2\right) \times 10^{-5}$ [7, Chapter 4].

Using the above values of Ω_M and $\Omega_{\rm rad}$, we conclude that the density of the cosmic medium is appreciably lower than the critical density, so that the Universe is open. This is the reason for using the metric (109) to describe the geometry of the Universe in the ΛCDM model. In our proposed S model, we also use the metric (109) and assume that the Universe is open.

The area of a surface reached by the photons emitted by a source with redshift z is determined by the relation

$$S(z) = 4\pi r^2(z), \tag{110}$$

where

$$r(z) = a_0 \sinh \chi(z), \tag{111}$$

and $a_0 = a(t_0)$ is the scale factor of the modern Universe. Taking cH_0^{-1} as a unit of length, we write $r(z)$ in the dimensionless form

$$\bar{r}(z) = r(z)/cH_0^{-1}. \tag{112}$$

The flux density of photons falling onto a receiver is proportional to $1/S(z)$. Due to the redshift, the energy of each detected photon $\hbar\omega_0$ differs from that of the emitted photon $\hbar\omega$. These energies are related as

$$\hbar\omega_0/\hbar\omega = a/a_0 = (1+z)^{-1}. \tag{113}$$

Obviously, the energy of each received photon is smaller than its energy when it is emitted by a factor of $(1+z)^{-1}$. In addition, the apparent brightness of an object with redshift z is diminished by a factor of $(1+z)^{-1}$. This is due to the fact that a unit time for the receiver corresponds to the time $(1+z)^{-1}$ for the emitter (see, e.g., [1, Chapter 3].

Taking into account all the above, the apparent brightness E of a source with absolute luminosity L and redshift z, neglecting photon absorption and scattering, can be written

$$E = L/[(1+z)^2 S(z)]. \tag{114}$$

Astronomers usually use magnitudes m instead of E. By definition,

$$m = -2.5 \log E + \text{const}. \tag{115}$$

To single out the effect of factors determining the Universe's evolution in the dependence $m(z)$ and to exclude the influence of the absolute luminosity of the observed object, we study objects having a known luminosity ("standard candles"). In addition to the apparent magnitude m, we introduce the notion of the absolute magnitude M. The quantity M is m for the case when the source is located at a distance of 10 pc from the observer. By definition,

$$M = -2.5 \log E_1 + \text{const}, \tag{116}$$

where $E_1 = L/4\pi l_0^2, l_0 = 10$ pc. With the expressions (112), (114), (115), and (116), we find

$$m - M = 5\log\left[(1+z)\bar{r}(z)\right] + 5\log\left(cH_0^{-1}/l_0\right). \quad (117)$$

The influence of factors determining the properties of the observed objects has been removed in the $(m-M)(z)$ dependence, and there only remains the dependence on the factors determining the Universe's evolution. We use (117) to calculate theoretically $(m-M)(z)$ in the ΛCDM model and in the model of a uniformly expanding Universe.

To find the function $\bar{r}(z)$ in (117), it is necessary to calculate $\chi(z)$ [see (111) and (112)]. The function $\chi(z)$ is unambiguously related to $a(t)$, which determines the dynamics of the Universe. For a photon moving towards a receiver located at the origin of the coordinate system χ, θ, φ,

$$ds^2 = c^2dt^2 - a^2(t)d\chi^2 = 0. \quad (118)$$

Hence, we find

$$d\chi = -cdt/a(t). \quad (119)$$

The minus sign is taken because we consider beams coming toward an observer located at the origin.

Using (108), we can pass from the variable t to the variable z, obtaining

$$dt^2 = a^2dz/a_0\left(da/dt\right), \quad (120)$$

and (119) can be written

$$d\chi = cdz/a_0\left(\dot{a}/a\right). \quad (121)$$

Thus, we obtain the function

$$\chi(z) = c\int\limits_0^z \frac{dz'}{a_0\left(\dot{a}/a\right)_{z'}}. \quad (122)$$

9.2. $(m-M)(z)$ Dependence in the ΛCDM Model

To calculate $(m-M)_\Lambda(z)$, we previously calculate the function $r(z)$. Taking into account (111), (112) and (91), we can write the relation determining the distance $r(z)$ to an observed object with redshift z in the form

$$\bar{r}_\Lambda(z) = \frac{\sinh\int\limits_0^z \frac{\sqrt{\Omega_{\rm curv}}dz'}{\sqrt{\Omega_{\rm curv}(1+z')^2 + \Omega_M(1+z')^3 + \Omega_{\rm rad}(1+z')^4 + \Omega_\Lambda}}}{\sqrt{\Omega_{\rm curv}}}. \quad (123)$$

Here and below, quantities calculated in the ΛCDM model will be denoted with the subscript Λ.

The parameters Ω_M, $\Omega_{\rm curv}$, $\Omega_{\rm rad}$ and Ω_Λ are not independent. It follows from (91) and (96) that

$$\Omega_{\rm curv} + \Omega_M + \Omega_{\rm rad} + \Omega_\Lambda = 1. \quad (124)$$

Most often observations are interpreted using the "flat ΛCDM model", in which it is assumed that $\Omega_{\rm curv} = 0$. The predictions of the ΛCDM model with appreciably nonzero $\Omega_{\rm curv}$ are not consistent with the observations (see, e.g., [7]). Assuming $\Omega_{\rm curv} = 0$, we write (123) and (124) in the form

$$\bar{r}_\Lambda(z) = \int\limits_0^z \frac{dz'}{\left[\Omega_M(1+z')^3 + \Omega_{\rm rad}(1+z')^4 + \Omega_\Lambda\right]^{1/2}}, \quad (125)$$

$$\Omega_M + \Omega_{\rm rad} + \Omega_\Lambda = 1. \quad (126)$$

If we agree with the assertion that space is flat and assume $\Omega_{\rm curv} = 0$, while the value of $\Omega_{\rm rad}$ is taken to be $\left(4.2/h^2\right) \times 10^{-5}$ (see, e.g., [7]), the parameters determining $\bar{r}_\Lambda(z)$ in the ΛCDM model are Ω_M and Ω_Λ, and, by virtue of (126), only one of these is independent, for example, Ω_M. In the ΛCDM model, the larger Ω_M, the larger the influence of gravitational and the smaller the influence of repulsive forces.

The standard mathematical procedure for identifying the theoretical $(m-M)_\Lambda(z)$ dependence that best describes the observational data on Type Ia supernovae yields $\Omega_M \approx 0.27$, $\Omega_\Lambda \approx 0.73$ (see, e.g., [7, 16–18]). Bearing this in mind, the following values of the parameter of the ΛCDM model are generally used:

$$\Omega_{\rm curv} = 0, \quad \Omega_M = 0.27, \quad (127)$$
$$\Omega_{\rm rad} = \left(4.2/h^2\right) \times 10^{-5}, \quad \Omega_\Lambda = 1 - \Omega_M - \Omega_{\rm rad},$$
$$H_0 = h \cdot 100 \text{ km/s Mpc}, \quad h = 0.7.$$

9.3. $(m-M)(z)$ Dependence in the S Model

With (122) and (102), Eq. (112) determining the distance $r(z)$ in the S model can be written

$$\bar{r}(z) = \gamma \sinh\left[\frac{1}{\gamma}\ln(1+z)\right]. \qquad (128)$$

Using (117) and (128), we obtain a formula for the $(m-M)(z)$ dependence in this model.

The S model contains the two independent parameters γ and h. Figure 3 plots the $(m-M)(z)$ dependences for the S model calculated using (117) for several values of γ. For comparison, a plot of the $(m-M)_\Lambda(z)$ dependence is also presented, calculated in the ΛCDM model for the parameter values (127).

A comparison of the theoretical $(m-M)(z)$ dependences presented in Fig. 3 calculated in the S and ΛCDM models shows the following. In the redshift range $z<2$, to each curve $(m-M)_\Lambda(z)$ of the ΛCDM model corresponding to particular values of Ω_M, $\Omega_{\rm rad}$, Ω_Λ, and h, we can match a curve $(m-M)(z)$ for the S model with parameters γ and h selected accordingly, which differs only little from the former curve. This means that the S model can explain the observational data on Type Ia supernovae as well as the ΛCDM model, at least at $z<2$ (precisely the range to which the observational data on Type Ia supernovae belong [17, 18]). As is clear from Fig. 3, the values of γ fort which the S model fits the observational data on Type Ia supernovae well are close to unity.

As Fig. 3 shows, the difference between the calculated curves $\bar{r}(z)$ and the plots $(m-M)(z)$ obtained in the S and ΛCDM models can become significant at redshifts $z>2$. To check the applicability of the S model to observations corresponding to $z \gg 1$, we use this model in the next section to explain the observed anisotropy of the CMB radiation. These observations correspond to redshifts $z \approx 1000$.

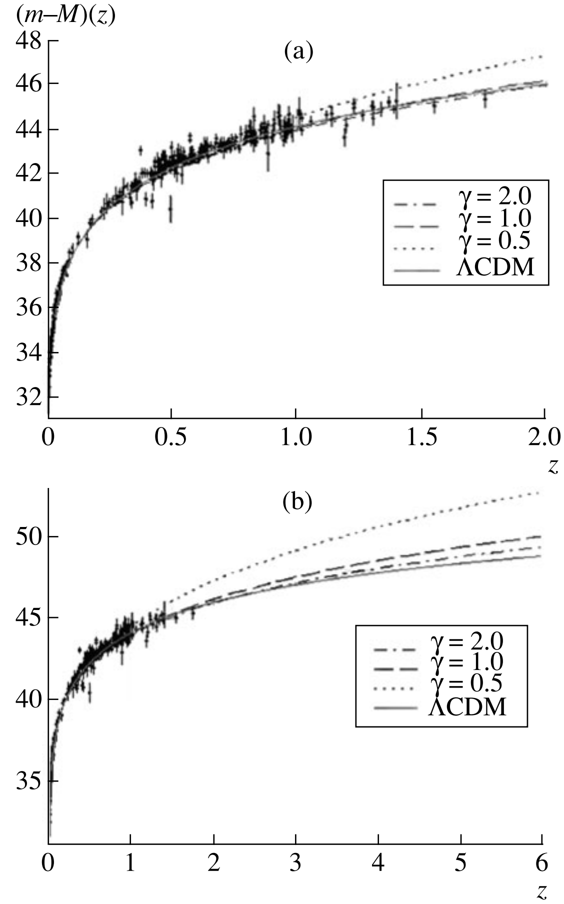

Fig. 3. $(m-M)(z)$ dependence in the S model for $h=0.7$ and various values of γ. For comparison, we also show the plot of $(m-M)_\Lambda(z)$ calculated for the ΛCDM model with the parameters (127). The ranges of z are: (a) $z \le 2$, (b) $z \le 6$. The points in the plots show the observed values of $(m-M)(z)$, and the vertical bars their observational uncertainties. The observational data are taken from [17, 18].

10. INTERPRETATION OF THE CMB ANISOTROPY

10.1. CMB anisotropy

In modern cosmology, it is believed that establishing smallness of the spatial curvature of the Universe (its "flatness") is a fundamental result of recent years [6, 7]. It is believed that data on anisotropy of the CMB convincingly demonstrate this result. These have been obtained as a result of vast systematic anisotropy observations with the aid of the Relict, COBE, and WMAP spacecraft [16–18, 29–31].

Analysis of the fine structure of the CMB shows that there are very small deviations from a uniform background. Weak temperature variations at the level of tens of ppm are observed. These indicate the existence of weak inhomogeneities (compressions and extensions) of the cosmic medium at the epoch of recombination. These inhomogeneities were seeds for galaxies and clusters of galaxies. In the compressions, the temperature was slightly higher than the mean value, and these are seen as bright spots (relative to the mean background). In the extensions, the temperature was slightly lower, and these are observed as relatively dark spots. The degrees of brightness deviation from the mean background differ, and vary from spot to spot, as well as among bright and dark spots.

Of special interest are the brightest spots in the CMB pattern. It is believed that the observed neighboring spots at the epoch of recombination of the cosmic medium are located at a well defined separation from each other. Following the structure formation theory based on Lifshitz's classical paper [32] (see also [33]), it is thought that this separation is specified by the age of the Universe at the epoch of recombination. This age turns out to be substantially different in the ΛCDM model and in the model of a uniformly expanding Universe. In the ΛCDM model, this ages is approximately 440 thousand years, whereas it is about 14 million years in the uniformly expanding model. In the ΛCDM model, the observed spots are thought to be protogalactic. In the S model, they are condensations that later gave rise to galaxy cluster formation.

The observations clearly indicate the existence of a certain angular separation $\Delta\theta$ between the spatial directions pointing at the centers of two neighboring spots.

The relation between the angular and linear sizes of an observed object depends on the geometry of space. To explain the observed angular separation $\Delta\theta$ in the ΛCDM model, we must assume that the space where the cosmological expansion occurs is almost flat and set $\Omega_{\text{curv}} \approx 0$. The size of the observed Universe is then approximately cH_0^{-1} (see, e.g., [6, 7]). It is believed that space can be regarded as flat on the scale cH_0^{-1}. Therefore, when solving the equations describing the dynamics of the Universe in the ΛCDM model, Ω_{curv} is usually set equal to zero. Disagreement with the observations arises for values of Ω_{curv} appreciably different from zero.

A fundamental difficulty of fitting the ΛCDM model to the observations is that, according to this model, the Universe expands non-uniformly most of the time. At the same time, the observations seem to indicate that the Universe expands uniformly.

We will show that we can explain the observed angular separation between the centers of neighboring bright spots against the homogeneous CMB background in the S model without invoking the idea of spatial flatness.

10.2. The Angular Size of Remote Objects

The formula for the angular extent $\Delta\theta$ of an object of linear size d at a redshift z can be written [7, Section 4.7]:

$$\Delta\theta = d(1+z)/r(z). \qquad (129)$$

In this formula, $r(z)$ is the distance to the observed object given by (111). Bearing in mind that the physical size of an object emitting photons at time t_i is $d = a(t_i)\sinh\chi$, the ratio $a(t_i)/a_0 = (1+z)^{-1}$ and the relation $r(z) = a_0 \sinh\chi$, and measuring lengths in units of cH_0^{-1}, we write (129) in the form

$$\Delta\theta = \bar{d}(1+z)/\bar{r}(z), \qquad (130)$$

where $\bar{d} = d/cH_0^{-1}$ and $\bar{r}(z) = r(z)/cH_0^{-1}$.

In the ΛCDM model, $\bar{r}(z)$ is calculated using Eq. (125). The ΛCDM model gives a correct value for the angular separation between the centers of neighboring bright spots against the homogeneous CMB background if Ω_M, Ω_{rad}, Ω_{curv}, Ω_Λ, and h are close to the values given in (127). The time t_{rec} when $a(t_{\text{rec}}) = 1/(1+z_{\text{rec}})$ in the ΛCDM model is found by solving (108) under the boundary conditions (112). Calculation shows that, with the parameter values (127),

$$t_{\text{rec}} \approx 4.4 \times 10^5 \text{ years}. \qquad (131)$$

In the model of a uniformly expanding Universe, the distance $\bar{r}(z)$ is calculated using (128). The recombination epoch t_{rec} in this model is given by the relation

$$t_{\text{rec}} = H_0^{-1}/(1+z_{\text{rec}}). \qquad (132)$$

Taking into account that $H_0^{-1} \approx 14 \times 10^9$ years and $z_{\text{rec}} \approx 1000$, we find that

$$t_{\text{rec}} \approx 14 \times 10^8 \text{ years}. \qquad (133)$$

Let us qualitatively explain what determines the characteristic size of the bright spots against the homogeneous CMB background.

There inevitably exist perturbations in an expanding Universe. When $t < t_{\text{rec}}$, the growth of these perturbations is prevented by the pushing-apart effect of the pressure of the relativistic component. After recombination, its influence on the dynamics of the non-relativistic component disappears. Therefore, the homogeneous distribution of the non-relativistic component becomes unstable.

At recombination, the density perturbations have a small magnitude, $\Delta\rho/\rho \sim 10^{-5}$ (see, e.g., [7]). Regions of increased density represent sources of an unbalanced gravitational field. Due to this field, contraction of non-relativistic matter located in causally connected regions, which by recombination had sizes of $l_{\text{rec}} = ct_{\text{rec}}$ or smaller, led to a growth of $\Delta\rho/\rho$ on these scales. Even an insignificant density increase over the mean density on sizes of $l \le l_{\text{rec}}$ in a uniformly expanding Universe is sufficient for inhomogeneities with sizes $l \le l_{\text{rec}}$ to become gravitationally bound. This is due to the fact that there is an exact balance between the attractive and repulsive forces in an unperturbed uniformly expanding Universe.

Beginning with the epoch of recombination, the non-relativistic component of the cosmic medium began to split into gravitationally bound clumps of size l_{rec} or smaller. Taking part in the common cosmological expansion, these fragments behave at $t \ge t_{\text{rec}}$ as quasi-independent gravitationally bound subsystems.

At present, in the uniformly expanding Universe inside the apparent horizon, there are about $(1+z_{\rm rec})^3 \sim 10^9$ inhomogeneities of characteristic size approximately equal to $l_{\rm rec} = ct_{\rm rec}$. Since $t_{\rm rec} \approx 14 \times 10^9$ years, we find $l_{\rm rec} \approx 4.6$ Mpc. This is the typical size of galaxy clusters in the modern Universe.

Note also that, in the S model, the time $t_{\rm eq}$ when the densities of the relativistic and non-relativistic components are equal is given by the relation

$$t_{eq} = H_0^{-1}\left(\rho_{20}/\rho_{10}\right) = H_0^{-1}\left(\Omega_{\rm rad}/\Omega_M\right). \quad (134)$$

If one admits that $\left(\frac{\Omega_{\rm rad}}{\Omega_M}\right) \approx \frac{4.2\times 10^{-5}}{\Omega_M h^2}$, then $t_{eq} \approx \frac{4.2\times 10^{-5} H_0^{-1}}{\Omega_M h^2}$. With $\Omega_M \approx 0.25$ and $h \approx 0.7$, we have $t_{eq} \approx 4.8 \times 10^6$ years. Since, in the model proposed, $t_{\rm rec} = 14 \times 10^6$ years and $t_{eq} = 4.8 \times 10^6$ years, we conclude that, at the epoch of recombination, the energy density of the cosmic medium was to a large extent determined by the non-relativistic component.

The size d determining the distance between neighboring spots against the homogeneous CMB background at the epoch of recombination is calculated as

$$d = 2ct_{\rm rec}. \quad (135)$$

With (135), we can write (129) in the form

$$\Delta\theta = \frac{2t_{\rm rec}\left(1+z_{\rm rec}\right)}{\bar{r}(z_{\rm rec})H_0^{-1}}\frac{180}{\pi}. \quad (136)$$

This relation gives the value of the angular separation in degrees.

In the ΛCDM model with the parameter values (127) and $z_{\rm rec} = 1000$, $t_{\rm rec} = 4.4 \times 10^5$ years, $\bar{r}_\Lambda(z) = 3.3$, and the value of this angular separation is $\Delta_\Lambda\theta = 1.09$. In the S model, $t_{\rm rec}\left(1+z_{\rm rec}\right) = H_0^{-1}$, so that

$$\Delta\theta = \frac{2\times 180}{\bar{r}(z_{\rm rec})\pi}. \quad (137)$$

With (128) and (137), we find that, in our proposed model, $\Delta\theta \approx 1^\circ$ if γ is taken to be 1.35. With this value of γ, the model of a uniformly expanding Universe also describes well the observed apparent magnitude—redshift relation for Type Ia supernovae in the redshift range $z \leq 2$ (Fig. 3).

When describing the dynamics of the Universe in the comoving reference frame, we should keep in mind the following. The quantity da/dt does not represent a physical velocity of any particles. There is no reason to think that da/dt cannot be larger than the speed of light. At the same time, the expansion rate $da/d\tau$ of the Universe in the Euclidean four-dimensional space is always lower than the speed of light. Let us verify this. Obviously,

$$da/dt = \gamma c, \quad c^2dt^2 = c^2d\tau^2 - da^2. \quad (138)$$

Hence, we find that

$$da/d\tau = c\frac{\gamma}{\sqrt{1+\gamma^2}}. \quad (139)$$

For any value of γ, we have $da/d\tau < c$. As γ increases, the quantity $da/d\tau$ monotonically grows. When $\gamma \gg 1$, we have $da/d\tau \approx c$.

11. CONCLUSION

(1) We have shown that the GR equations can be supplemented not only with Einsteinian repulsive forces, described by the Λ term, but also with other forces. We have presented the generalized Einstein equations allowing for the existence of such forces.

(2) We have considered in detail the theoretically admissible case that the cosmological repulsive and attractive forces precisely balance each other. A model of a uniformly expanding Universe (S model) has been proposed.

(3) We have written cosmological equations describing the S model. In the comoving coordinate system, these have the form

$$\frac{da}{dt} = \gamma c, \quad \frac{d^2a}{dt^2} = 0,$$

where γ is the S model parameter that determines the rate of variation of the spatial scale $a(t)$ (the radius of curvature of the Universe). In addition to the parameter γ, the dynamics of the Universe in the S model depends on the value of the Hubble constant H_0.

(4) It has been shown that the equations determining the evolution of $a(t)$ in the S and ΛCDM models are fundamentally different, while the equations relating $a(t)$ and the parameters determining the thermodynamic properties of the cosmic medium are the same in these models.

(5) An interpretation of the apparent magnitude—redshift relation $(m-M)(z)$ for Type Ia supernovae is given in the S model, which can describe these data well. The difference between the calculated $(m-M)(z)$ curves obtained in the S and ΛCDM models can be substantial at redshifts $z > 2$.

(6) We can explain the observed angular separation between the centers of neighboring bright spots $(\Delta\theta \approx 1^\circ)$ against the homogeneous CMB background using the S model, without assuming spatial flatness. The values of γ and h for which $\Delta\theta \approx 1^\circ$ and the calculated $(m-M)(z)$ curve describes well the

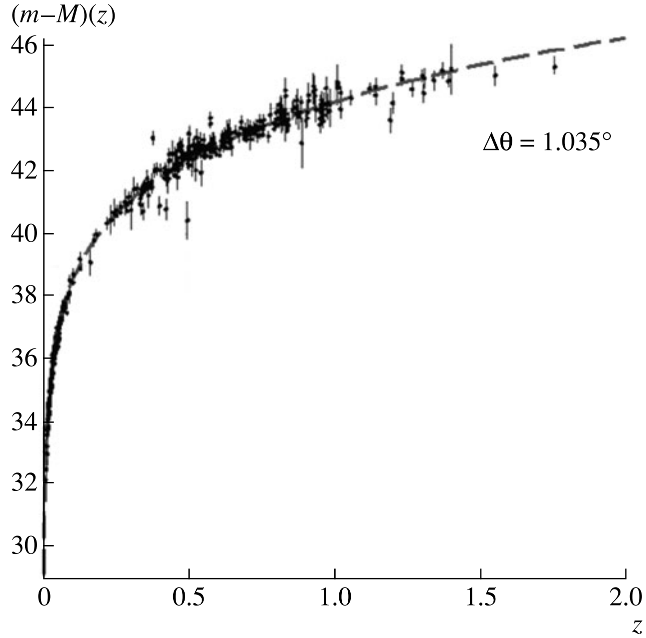

Fig. 4. $(m - M)(z)$ dependence in the S model for $\gamma = 1.22$ and $h = 0.63$. We give the value of the angular separation between CMB spots for these values of the parameters.

observations for Type Ia supernovae turn out to be $\gamma = 1.22$, $h = 0.63$ (Fig. 4).

(7) In the S model, the lifetime of the Universe is determined by the value of the Hubble constant, and is precisely equal to H_0^{-1}. With $H_0 = 63\ \mathrm{km\,s^{-1}Mpc^{-1}}$, this implies approximately 15.5×10^9 years, consistent with modern views of the age of the Universe.

(8) Due to its simplicity, the S model makes it possible to improve our understanding of the laws governing the evolution of the Universe. It does not contain singularities in the behavior of the scale factor, such as occur in other cosmological models. According to the S model, the dates of the most important processes in the early Universe are completely different from the predictions of modern cosmology. For example, the duration of the high-temperature epoch that is favorable for nucleosynthesis is much longer in this model than in the ΛCDM model. Therefore, the ratio between the amounts of hydrogen and heavier elements in the primordial chemical composition of the cosmic medium may prove to be different from the conventional ones. In the present paper, we have not applied the S model to explain the early evolution of the Universe.

In conclusion, we note that the character of the expansion of the Universe at $z > 1000$ may differ from what is proposed in this paper. At the same time, applying the S model to interpret observations at $z \leq 1000$ shows that the assumption of a uniformly expanding Universe in the post-recombination period seems to be correct.

The possibility of carrying out computer simulations of the dynamics of the Universe that enable independent variation of the model parameters with a clear graphical representation of the calculated results for various cosmological models, including the S model, is available on our site, http://www.cosmoway.ru/.

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REFERENCES

1. Ya. B. Zel'dovich and I. D. Novikov, *The Structure and Evolution of the Universe* (Nauka, Moscow, 1975; Univ. of Chicago, Chicago, 1983).
2. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972; Platon, Moscow, 2000).
3. S. Perlmutter, G. Aldering, G. Goldhaber, et al., Astrophys. J. **517**, 565 (1999).
4. A. G. Riess, A. V. Filippenko, P. Challis, et al., Astron. J. **116**, 1009 (1998).

5. E. B. Glinner, Usp. Fiz. Nauk **172**, 221 (2002) [Phys. Usp. **45**, 213 (2002)].
6. A. D. Chernin, Usp. Fiz. Nauk **178**, 267 (2008) [Phys. Usp. **51**, 253 (2008)].
7. D. S. Gorbunov and V. A. Rubakov, *Introduction to the Theory of the Early Universe. Hot Big Bang Theory* (URSS, Moscow, 2008) [in Russian].
8. P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988).
9. P. J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).
10. U. Alam, V. Sahni, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. **02**, 011 (2007).
11. V. A. Rubakov, Phys. Rev. D **61**, 061501 (2000).
12. A. D. Chernin, D. I. Santiago, and A. S. Silbergleit, Phys. Lett. A **294**, 79 (2002).
13. M. P. Dabrowski and T. Stachowiak, Ann. Phys. (N.Y.) **321**, 771 (2006).
14. M. Sami and A. Toporensky, Mod. Phys. Lett. A **19**, 1509 (2004).
15. M. Libanov, V. Rubakov, E. Papantonopoulos, et al., J. Cosmol. Astropart. Phys. **08**, 010 (2007).
16. D. N. Spergel, R. Bean, O. Dore, et al., Astrophys. J. Suppl. Ser. **170**, 377 (2007); arXiv:astro-ph/0603449 (2006).
17. P. Astier, J. Guy, N. Regnault, et al., Astron. Astrophys. **447**, 31 (2006).
18. A. G. Riess, L. G. Strolger, S. Casertano, et al., Astrophys. J. **656**, 98 (2007); e-Print arXiv: astro-ph/0611572 (2006).
19. V. N. Lukash and V. A. Rubakov, Usp. Fiz. Nauk **178**, 301 (2008) [Phys. Usp. **51**, 283 (2008)].
20. A. M. Cherepashchuk and A. D. Chernin, Byull. RAN Zashch. Nauki No. 4, 177 (2008).
21. V. A. Klimenko and A. M. Fridman, *The Centrifugal Nature of Dark Energy*, Preprint IAE-6597/1 (RNTs Kurchat. Inst., Moscow, 2009) [in Russian].
22. A. Einstein, *Principles of General Theory of Relativity*, in *Collection of Sci. Works*, vol. 1 (Henry Holt, New York, 1920; Nauka, Moscow, 1965).
23. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics,* Vol. 2: *The Classical Theory of Fields* (Nauka, Moscow, 1988; Pergamon, Oxford, 1975).
24. A. A. Fridman, Usp. Fiz. Nauk **80**, 439 (1963).
25. A. A. Fridman, Usp. Fiz. Nauk **80**, 448 (1963).
26. A. Einstein, *Cosmology Questions and General Theory of Relativity*, in *Collection of Sci. Works*, vol. 1 (Nauka, Moscow, 1965) [in Russian].
27. A. Sandage, G. A. Tammann, A. Saha, et al., Astrophys. J. **653**, 843 (2006); arXiv: astro-ph/0603647 (2006).
28. D. N. Spergel, L. Verde, H. V. Peiris, et al., Astrophys. J. Suppl. Ser. **148**, 175 (2003).
29. G. Hinshaw, M. R. Nolta, C. L. Bennett, et al., Astrophys. J. Suppl. Ser. **170**, 377 (2007).
30. A. M. Cherepashchuk and A. D. Chernin, *Universe, Life, Black Holes* (Vek-2, Fryazino, 2003) [in Russian].
31. G. Hinshaw, M. R. Nolta, C. L. Bennett, et al., arXiv: astro-ph/0603451 (2006).
32. E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **16**, 587 (1946).
33. E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk **80**, 391 (1963) [Sov. Phys. Usp. **6**, 495 (1963)].

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